

Towards Sub-Nyquist Tissue Doppler Imaging Using Non-Uniformly Spaced Stream of Pulses

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Abstract— Tissue Doppler ultrasound imaging enables quantification of the heart function by mapping the velocity of the tissue in different regions within the left ventricle. This estimation is performed by transmitting a series of evenly spaced pulses at a specific direction and analyzing phase shifts in the returning echoes. Traditionally the received signals are sampled at the Nyquist rate. However, high sampling rates produce high volumes of data. In addition, the number of consecutive pulses needed for reliable velocity estimation at each orientation in the presence of clutter limits the size of the sectors that can be imaged simultaneously. In this work, compressed sensing techniques that were suggested for solving similar problems in radar signal processing are adapted to tissue Doppler ultrasound imaging. The proposed method reduces the amount of data propagated through the scanner in two ways: A non-uniformly spaced stream of pulses is transmitted and each received echo is sampled at a sub-Nyquist rate. In addition, the time needed for each velocity estimation is reduced. The proposed method was evaluated using realistic synthetic echocardiographic sequences.

Keywords— *compressed sensing (CS), ultrasound, Xampling, tissue Doppler imaging (TDI)*

I. INTRODUCTION

Tissue Doppler ultrasound imaging (TDI) enables estimation of local velocities of the cardiac tissue by transmitting a stream of pulses in a certain direction and estimating the velocity of the tissue from the phase shifts of the returning echoes. These estimates are used in order to quantitatively assess the cardiac function by calculating, e.g., the localized myocardial strain and strain rate. The time needed for each single velocity estimation is the coherent processing interval (CPI). In order to estimate the velocity of the tissue precisely and separate slow tissue movement from dominant clutter, a large number of consecutive pulses has to be transmitted in the same direction. The number of transmitted pulses per unit of time is limited by the speed of sound and the desired depth being examined. Therefore, there is an inherent tradeoff between spectral and spatial resolution. This limitation impedes spectral Doppler usage to a few measurements through the LV wall during the cardiac cycle.

Two of the leading trends in the evolution of ultrasound scanners in recent years are the increasing amount of elements in ultrasound probes (in particular three-dimensional probes) and the emergence of hand-held/wireless systems. As the amount of transducer elements increases so does the amount

of data that has to be processed and stored. On the other hand, hand-held/wireless systems have lower resources and have to cope with limited bandwidth while producing high-quality images. As a result of these trends there is a rising interest in the past few years in reducing the amount of data propagated through the scanner and performing the reduction as close as possible to the analog front end.

Compressed sensing (CS) is a signal processing framework that facilitates the reconstruction of signals sampled at a sub-Nyquist rate using priors on their sparsity [1], [2]. Combining CS with classic sampling theorems has been the basis for Xampling, a framework that facilitates the reconstruction of real analog signals sampled at sub-Nyquist rates [3]. In [4], the authors suggested to model ultrasound signals detected by the transducer following the transmission of a single pulse as a sum of a relatively small number of pulses, replicas of the scanner's PSF. This model was later extended in [5] and [6] to incorporate the beamforming process performed on sub-Nyquist sampled element data. Similar methods were used for the processing of radar signals and the detection of both the location and the velocity of the targets while sampling the received signal following each transmission (fast time signal) at a sub-Nyquist rate [7], [8]. Recently, it was suggested that the number of pulses needed for each velocity estimation (slow time) could be effectively reduced by transmitting non-uniformly spaced stream of pulses [13].

In this work we adopt the approach of [7] and [13] for sampling and processing of Doppler signals and apply it to TDI. By processing the signals within the CS framework, this work aims to facilitate reduced number of pulses per velocity estimation and reduced fast time sampling rate. In classic spectral Doppler processing schemes each velocity estimation requires the transmission of M consecutive pulses. The number of pulses determines the spectral resolution of the measurement. Therefore the entire CPI is used to produce a single velocity estimation. Using the proposed method only a certain percentage of the pulses within each CPI are used for estimation of the velocity profile in each direction allowing the scanner to switch between several imaging orientations and potentially map a sector of the imaged space simultaneously by using the resulting time gaps. The reduced fast time sampling rate reduces the amount of data propagated in the system and can be used for both high-end systems with

probes containing many elements as well as for handheld/wireless systems.

II. PARAMETRIC MODEL FOR TDI SIGNALS

First, consider an ultrasound transducer emitting a focused pulse train at a certain direction. When M consecutive and equally spaced pulses are transmitted (as in the current technology) the transmitted signal is:

$$x_T(t) = \sum_{p=0}^{M-1} h\left(t - pT_{prf}\right) \sin\left(2\pi f_0\left(t - pT_{prf}\right)\right), \quad 0 \leq t \leq MT_{prf}, \quad (1)$$

where $h(t)$ is the envelop of the transmitted narrowband pulse with Fourier transform $H(\omega)$ and T_{prf} is the pulse repetition interval. Next, a parametric model describing the TDI data in both fast-time (A-line) and slow-time (between pulses) is proposed. Later this model will be extended to the case in which a non-uniformly spaced stream of pulses is used while preserving the overall CPI. In this work the scatterers are defined in the radial coordinate system. Extending the model in [9] to L scatterers, the model for the received IQ signal assumed in this work is:

$$x_R(t) = \sum_{p=0}^{M-1} \sum_{l=1}^L \bar{a}_l \cdot h\left(t \left(1 - \frac{2v_{z,l}}{c}\right) - \frac{2d_l}{c} - pT_{prf} \left(1 + \frac{2v_{z,l}}{c}\right)\right) \cdot \exp\left(-j2\pi f_0 \frac{2v_{z,l}}{c} T_{prf} p\right), \quad (2)$$

Here L is the number of dominant scattering elements contained in each A-line, c is the speed of sound in soft tissue and f_0 is the central frequency of the transmitted pulse. In this model each dominant scatterer is defined by three parameters: its complex amplitude \bar{a}_l (reflecting its scattering strength), its depth d_l (reflecting distance from the transducer) and its radial velocity $v_{z,l}$.

In order to simplify the model, an assumption on the relative velocity of the scatterers was made. Specifically, we assume "slow target" meaning that the velocity of the scatterer is negligible compared to the speed of sound in the medium. In addition, under this assumption, the depth of each target can be considered to be the same during the CPI. The assumption of slow targets is reasonable in TDI since the speed of sound in soft tissue is 1540 m/sec while the maximal expected left ventricle velocity is around 10 cm/s [10]. Assuming "slow targets" leads to the following model for tissue Doppler signals:

$$x_R(t) = \sum_{p=0}^{M-1} \sum_{l=1}^L \bar{a}_l \cdot h\left(t - \tau_l - pT_{prf}\right) \cdot \exp\left(-jv_l T_{prf} p\right) \quad (3)$$

$$\tau_l \equiv \frac{2d_l}{c}, \quad v_l \equiv 2\pi f_0 \frac{2v_{z,l}}{c}.$$

here v_l is the Doppler frequency of the dominant scatterer l and τ_l is its delay. Therefore the received signal from L dominant scatterers is completely defined by the 3L degrees of freedom (DOF): $\{\bar{a}_l, \tau_l, v_l\}_{l=1}^L$ of the L scattering elements.

In spectral Doppler ultrasound, where a single angle and depth are monitored, the velocity at the desired depth is estimated using M -point DFT and displayed as a short time Fourier transform plot. The spectral resolution obtained using this method is $2\pi/MT_{prf}$, proportional to the length of the CPI. The goal of this work is to accurately estimate the imaged velocity field while sampling at sub-Nyquist sampling frequencies, by estimating the 3L DOF of the L scattering elements. The implementation of this scheme will be discussed in the next section.

III. COMPRESSED SENSING TISSUE DOPPLER IMAGING

In order to process the received signals and estimate the Doppler frequencies, the received A-lines are aligned, normalizing the beginning of each A-line to the start of the matching transmitted pulse. Following alignment, a single A-line $x_p(t)$ can be written as

$$x_p(t + pT_{prf}) = \sum_{l=1}^L \bar{a}_l \cdot h(t - \tau_l) \cdot \exp\left(-jv_l T_{prf} p\right). \quad (4)$$

The Fourier series representation of a single A-line is

$$X_p[k] = \frac{1}{T_{prf}} H[k] \sum_{l=1}^L \bar{a}_l \cdot \exp\left(-j2\pi k \tau_l / T_{prf}\right) \cdot \exp\left(-jv_l T_{prf} p\right). \quad (5)$$

Traditionally, the received signals are sampled at the Nyquist rate, which is equivalent to using all the Fourier coefficients within the bandwidth of the transducer. Exploiting priors on the sparsity of the signal, only a certain fraction of these coefficients were used in this work, achieving sub-Nyquist fast-time sampling. Following [13], suppose that a non-uniform pulse train is used and the p^{th} pulse is transmitted at time $t = m_p T_{prf}$ where $m_p \geq p$ and $\{m_p\}_{p=0}^{P-1}$ is an ordered set of integers. In this case, the transmitted signal takes on the form:

$$x_T(t) = \sum_{p=0}^{P-1} h\left(t - m_p T_{prf}\right) \sin\left(2\pi f_0\left(t - m_p T_{prf}\right)\right). \quad (6)$$

Accordingly, a single A-line received by the scanner could be written following alignment as

$$x_p(t + m_p T_{prf}) = \sum_{l=1}^L \bar{a}_l \cdot h(t - \tau_l) \cdot \exp\left(-jv_l T_{prf} m_p\right) \quad (7)$$

and its Fourier series as

$$X_p[k] = \frac{1}{T_{prf}} H[k] \sum_{l=0}^{L-1} \bar{a}_l \cdot \exp(-j2\pi k \tau_l / T_{prf}) \cdot \exp(-j\nu_l T_{prf} m_p). \quad (8)$$

In order to estimate the 3L parameters from the received measurements, the range and frequency of each dominant scatterer are selected from a discrete grid. Following the notations of [13] the integers $0 \leq r_l \leq M-1$ satisfy $2\pi r_l / M = \nu_l T_{prf}$ and the integers $0 \leq s_l \leq N-1$ satisfy $s_l / N = \tau_l / T_{prf}$.

Now, the system of equations of (8) can be written in matrix form as

$$X = H F_N^K A (F_M^P)^T \quad (9)$$

where X is the $K \times P$ measurement matrix with elements $X(k, p) = X_p[k]$. H is a diagonal matrix defined by the PSF of the system $H = \text{diag}(H[k]) / T_{prf}$. The matrices F_N^K and F_M^P are partial DFT matrices, where F_N^K contains K out of the N rows of the full $N \times N$ DFT matrix and F_M^P contains P out of the M rows in the full $M \times M$ DFT matrix. For sub-Nyquist sampling in both the fast and the slow time, let $K < N$ and $P < M$. The matrix A contains the 3L parameters in the following way: each non-zero element defines the depth r_l and velocity s_l of a dominant scatterer l . The amplitude of the element is the estimated amplitude of the scatterer. By compensating for the effect of the PSF on the samples and considering

$$Y = XH^{-1} \quad (10)$$

the following CS problem is derived

$$Y = F_N^K A (F_M^P)^T. \quad (11)$$

The sparse representation matrix A is estimated from the matrix Y using the matrix form of the fast iterative shrinkage threshold algorithm [11] solving the l_1 minimization problem:

$$\min_A \left\{ \frac{1}{2} \|Y - F_N^K A (F_M^P)^T\|_F^2 + \lambda \|A\|_1 \right\}, \quad (12)$$

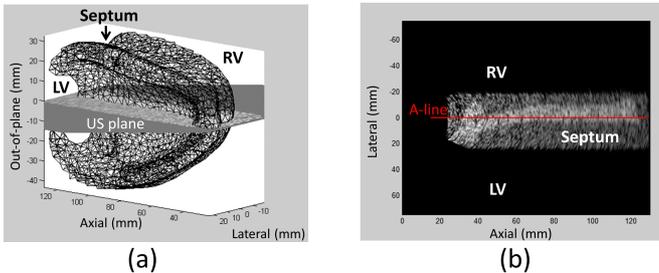


Fig. 1. Realistic synthetic TDI phantom based on bio-mechanical model

where λ is a regularization parameter. Since TDI estimates the velocity of the cardiac tissue at a small region and does not attempt to evaluate the velocity of each dominant scatterer, the estimated range-velocity matrix is then convolved with the PSF of the system, similar to [6].

IV. VALIDATION

Synthetic TDI data was used in order to evaluate the algorithm and provide realistic ground truth motion fields. Data was simulated with a variation of the pipeline in [12]. In these simulations, the geometries of the left and right ventricles are extracted from a clinical 3D ultrasound sequence (fig 1. a), while the synthetic motion is obtained by applying a bio-mechanical model to the segmented geometry. The motion model was applied to a set of point scatterers, from which ultrasound data was synthesized by applying the ultrasound simulator COLE (fig. 1 b). The pipeline was used to simulate TDI data from a scan line crossing the septum longitudinally, as done clinically to measure, e.g., longitudinal strain/strain-rate (e.g. [10]), cf. fig 1. b.

The simulated dataset includes a single heartbeat imaged at a pulse repetition frequency of 2000Hz. The simulated pulse had a central frequency of 3MHz and a Gaussian envelope with a full width at half maximum of 2 μ s. The pulse trains used in this work are directed to the center of the septum and the full reference pulse stream included $M = 50$ pulses per velocity estimation. Using the proposed processing scheme, subsets of 20%-80% of the received pulse-echo transmissions were randomly selected. From these pulse-echo subsets, spectral Doppler estimates were produced and compared to the estimations based on the full data set. Fast-time sampling rate of only 40% of the Nyquist frequency was used throughout this work in order to stay compatible with [5] and [6], selecting the K Fourier coefficients closest to the resonance frequency of the transducer.

V. RESULTS

The representation matrices (A in (11)) produced by the algorithm for each CPI present the locations of dominant scatters along the A-lines and their corresponding velocities (fig 2.). The depth-velocity map produced using 50% of the pulses matches the depth-velocity map produced from the full pulse train (fig 2. a and b). Moreover, the depth-velocity map produced from only 20% of the pulses present a similar image, although small aliasing related artifacts can be seen (fig 2. c). It should be noted that for a functional modality as TDI the precise location of each dominant scatterer is less important

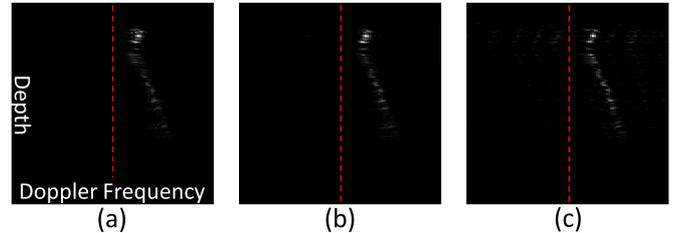


Fig. 2. Depth-velocity maps produced from 100% (a), 50% (b) and only 20% (c) of the pulses

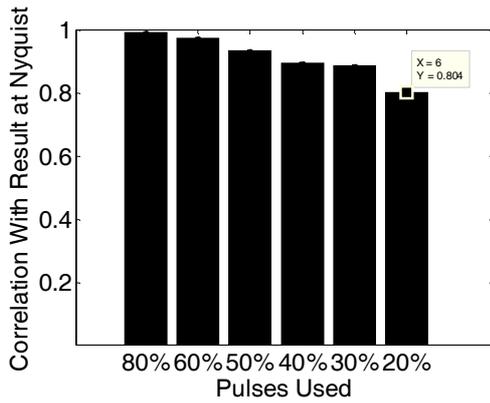


Fig. 3. Correlation coefficient R between the estimated velocities calculated from the partial and the full pulse trains

than the correct estimation of the velocity profile as a function of depth and this estimation is stable even when a small percentage of the pulses are used.

In order to quantitatively assess the performance of the algorithm the correlation was evaluated between the depth-velocity maps produced using different percentages of the pulses and the one produced from the full dataset. The correlation coefficient R between the estimated velocities calculated from 50% of the pulses and those calculated from the entire dataset is 0.95. When fewer pulses are used, this correlation gradually decreases (fig. 3). However, with only 20% of the pulses this correlation is still as high as 0.8.

The pulses transmitted during a heartbeat can be divided into a series of partially overlapping CPIs. Spectral estimations from specific depths can be concatenated to produce spectral Doppler images equivalent to those produced using classic processing schemes. In fig. 4 a series of such spectral Doppler estimations taken from a depth of 6.2cm are presented. The plots show typical S' (i.e. systolic shortening) and E' (i.e. diastolic motion) waves. These estimations were produced from 100%, 50% and 30% of the pulses (fig. 4 a, b and c respectively). Similar to the depth-velocity maps, the estimates produced from 100% and 50% of the pulses are similar while the spectral Doppler map produced from 30% of the samples presents the same velocity profile with weak aliasing noise.

VI. CONCLUSION

In this work a parametric model for TDI signals was proposed along with a framework for the estimation of spectral Doppler maps from ultrasound signals sampled at a sub-Nyquist rate in both fast and slow time: only a subset of the fast time frequencies were used and non-uniformly spaced stream of pulses were transmitted. This non-uniform pulse stream preserves the total CPI while reducing the time needed for the estimation of the velocity profile at a certain orientation. The current performance of the proposed algorithm is satisfactory, and could still be improved since the pulse selection was not optimized. The proposed parametric signal model was supported by our results, validating the assumptions on "slow targets" and the ability to cope with

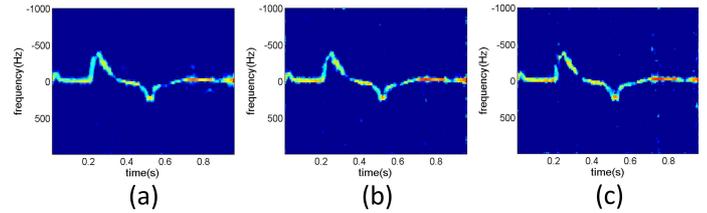


Fig. 4. Spectral Doppler estimations produced from 100% (a), 50% (b) and 30% (c) of the pulses

accelerating tissue. The compressed sensing framework for TDI presented in this work reduces the number of pulses needed for velocity estimation at a given spectral resolution, changing the tradeoff between spectral and spatial resolution. The remaining time between pulses in each direction can be used for scanning in different directions during the same CPI. Future research directions include: extension of the proposed method to enable color Doppler and vector Doppler estimations and the optimization of the reconstruction algorithm and the selection of transmitted pulses.

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